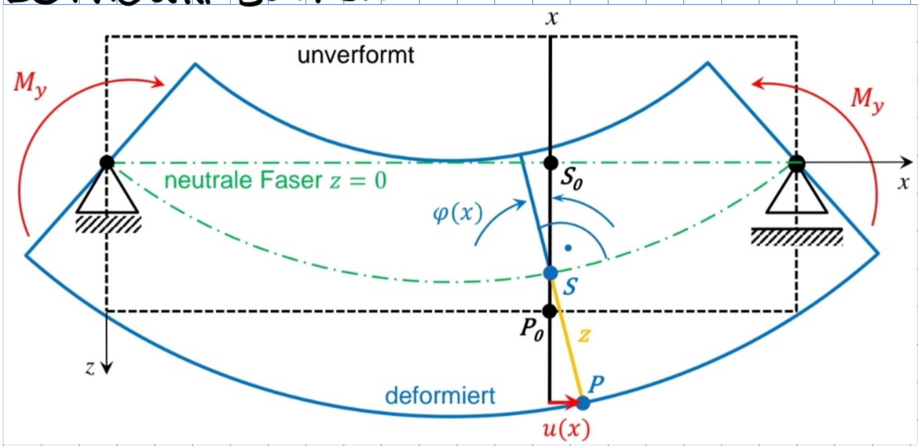
FL 2 Zusammenfassung

$1\,MPa = 1\,N/mm^2$

Bernoulli Balken



Kinematische Annahmen:

- Keine Deformation in y-Richtung
- Querschnitte bleiben eben und senkrecht auf Mittellinie

Flächenträgheitsmomente

Mass für den Widerstand des QS gegen Biegung

Bei $z=0$ → neutrale Faser $[mm^4]$

$\boxed{\int_A z\,dA = 0}$ FTMe $= [mm^4]$

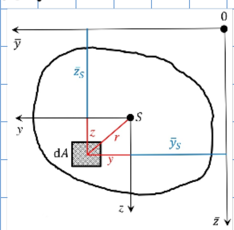
$\boxed{\int_A z\,dA = A z_s \text{ FTM 1.Ord}}$

$\boxed{I_{yy} = \int_A z^2\,dA \text{ FTM 2.Ord.}}$

Fläche	I_y	I_z	I_{yz}	I_p	$I_{\bar{y}}$
Rechteck	$\frac{bh^3}{12}$	$\frac{hb^3}{12}$	0	$\frac{bh}{12}(h^2+b^2)$	$\frac{bh^3}{3}$
Quadrat (Gleich wenn a Gedreht)	$\frac{a^4}{12}$	$\frac{a^4}{12}$	0	$\frac{a^4}{6}$	$\frac{a^4}{3}$
Dreieck	$\frac{bh^3}{36}$	$\frac{bh}{36}(b^2-ba+a^2)$	$-\frac{bh^2}{72}(b-2a)$	$\frac{bh}{36}(h^2+b^2-ba+a^2)$	$\frac{bh^3}{12}$
Kreis	$\frac{\pi d^4}{64} = \frac{\pi R^4}{4}$	$\frac{\pi R^4}{4}$	0	$\frac{\pi R^4}{2}$	$\frac{5\pi}{4}R^4$
dünner Kreisring $t << R_m$	$\pi R_m^3 t$	$\pi R_m^3 t$	0	$2\pi R_m^3 t$	$3\pi R_m^3 t$
Halbkreis	$\frac{R^4}{72\pi}(9\pi^2-64)$	$\frac{\pi R^4}{8}$	0	$\frac{R^4}{36\pi}(9\pi^2-32)$	$\frac{\pi R^4}{8}$
Ellipse	$\frac{\pi}{4}ab^3$	$\frac{\pi}{4}ba^3$	0	$\frac{\pi ab}{4}(a^2+b^2)$	$\frac{5\pi}{4}ab^3$

Satz von Steiner

$$I_{\bar{y}\bar{y}} = \bar{z}_s^2 A + I_{yy}$$
$$I_{\bar{z}\bar{z}} = \bar{y}_s^2 A + I_{zz}$$
$$I_{\bar{y}\bar{z}} = -\bar{y}_s \bar{z}_s A + I_{yz}$$



$I_{yy} = \int_A z^2 dA > 0$	axiales FTM
$I_{zz} = \int_A y^2 dA > 0$	axiales FTM
$I_{yz} = -\int_A yz\,dA <> 0$	gemischtes FTM
$I_p = \int_A (y^2+z^2)dA = \int_A r^2 dA = I_{yy} + I_{zz} > 0$	polares FTM

Parallelverschiebung

Steiner immer addieren

$$I_{\bar{y}\bar{y}} = \bar{z}_S{}^2 A + I_{yy}$$
$$I_{\bar{z}\bar{z}} = \bar{y}_S{}^2 A + I_{zz}$$
$$I_{\bar{y}\bar{z}} = -\bar{y}_S \bar{z}_S A + I_{yz}$$

Flächenträgheitstensor

$$I = \begin{pmatrix} I_{yy} & I_{yz} \\ I_{zy} & I_{zz} \end{pmatrix} \qquad A = \begin{pmatrix} \cos(\varphi) & \sin(\varphi) \\ -\sin(\varphi) & \cos(\varphi) \end{pmatrix}$$

$$\boxed{I' = A \cdot I \cdot A^T} \qquad \boxed{A^{-1} = A^T}$$

$$I_{y'y'} = \frac{1}{2}(I_{yy} + I_{zz}) + \frac{1}{2}(I_{yy} - I_{zz})\cos 2\varphi + I_{yz}\sin 2\varphi$$

$$I_{z'z'} = \frac{1}{2}(I_{yy} + I_{zz}) - \frac{1}{2}(I_{yy} - I_{zz})\cos 2\varphi - I_{yz}\sin 2\varphi$$

Deviationsmoment

$$I_{y'z'} = -\frac{1}{2}(I_{yy} - I_{zz})\sin 2\varphi + I_{yz}\cos 2\varphi$$

Drehwinkel vom xy- ins Hauptachsensystem

$$\tan 2\varphi^* = \frac{2I_{yz}}{I_{yy} - I_{zz}} \qquad \boxed{\varphi^* = \frac{1}{2}\arctan\left(\frac{2I_{yz}}{I_{yy} - I_{zz}}\right)}$$

Hauptmomente

$$I_{1/2} = \frac{I_{yy} + I_{zz}}{2} \pm \sqrt{\frac{(I_{yy} - I_{zz})^2}{4} + I_{zy}^2}.$$

Biegespannung

$$\boxed{\sigma_b(x,y) = \frac{M_y(x)}{I_{yy}(x)} \cdot z} = \frac{M_B}{W_B}$$

$$\boxed{\sigma_{max} = \frac{M}{I} \cdot z_{max}}$$

$$\boxed{\sigma_{min} = \frac{M}{I} \cdot z_{min}}$$

$$\boxed{I_{erf} = \frac{M}{\sigma_{zul}}}$$

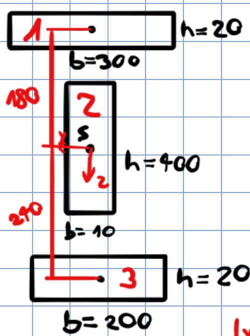
Widerstandsmoment

$$\boxed{W = \frac{I}{z}}$$

I: Flächenträgheitsmoment

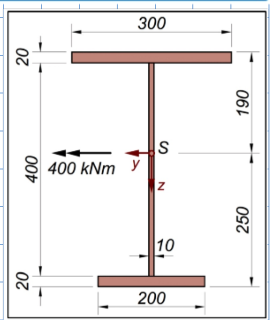
z: maximaler Randfaserabstand

Flächenträgheitsmomente um y-Achse berechnen



$$\boxed{I_y = \frac{bh^3}{12}}$$

$$I_{ges} = I_1 + I_2 + I_3$$

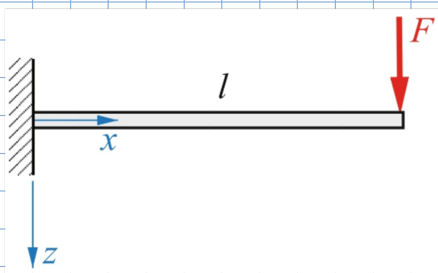


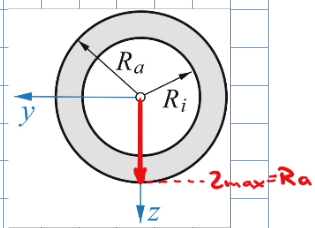
	I_y	+ $z^2 \cdot A$
①	$\frac{300 \cdot 20^3}{12}$	$+ 180^2 \cdot 300 \cdot 20$
②	$\frac{10 \cdot 400^3}{12}$	–
③	$\frac{200 \cdot 20^3}{12}$	$+ 240^2 \cdot 200 \cdot 20$

$$= \underline{\underline{478466666 \text{ mm}^4}}$$

Bsp. Gesucht ist F_{max} für σ_{zul}

Geg: $R_a = 5\text{cm}$, $R_i = 4\text{cm}$, $l = 3\text{m}$, $\sigma_{zul} = 150 \text{ MPa}$





$$M = l \cdot F$$

$$\sigma_{zul} = \frac{M}{W} = \frac{l \cdot F}{W}$$

$$W = \frac{I}{z_{max}}$$

I: Flächenträgheitsmoment

z_{max}: maximaler Randfaserabstand

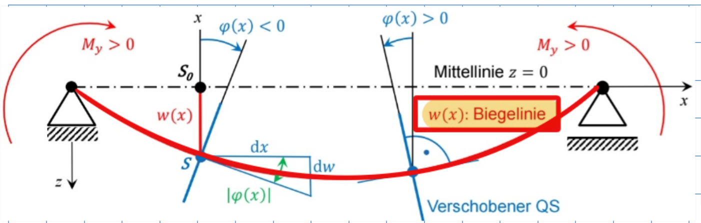
$$I_y = \frac{\pi (R_a^4 - R_i^4)}{4}$$

$$\rightarrow F_{max} = \frac{\sigma_{zul} \cdot W}{l} = \frac{\sigma_{zul} \cdot I}{l \cdot z_{max}} = \frac{\sigma_{zul} \cdot \frac{\pi (R_a^4 - R_i^4)}{4}}{l \cdot z_{max}}$$

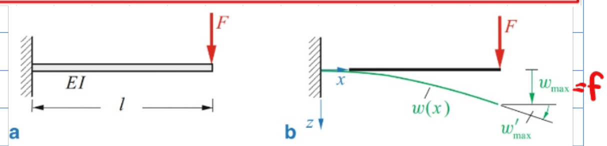
$$= \frac{150 \text{ MPa} \cdot \frac{\pi (5\text{cm}^4 - 4\text{cm}^4)}{4}}{3\text{m} \cdot 5\text{cm}} = \underline{\underline{2898.12 \text{ N}}}$$

Gesamt Schwerpunkt

$$x_S = \frac{A_1 \cdot x_1 + A_2 \cdot x_2}{A_1 + A_2}, \quad y_S = \frac{A_1 \cdot y_1 + A_2 \cdot y_2}{A_1 + A_2}$$

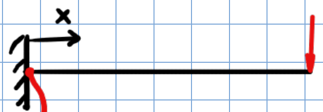


$w(x)$	⟷ Durchbiegung	$\int dx$	
$w'(x)$	⟷ Neigung		Einstiegspunkt, wenn Balken
$w''(x)$	⟷ Biegemoment		← stat. bestimmt gelagert, d.h. $M_b(x)$ kann
$w'''(x)$	⟷ Querkraft		über GGB berechnet werden
$w^{(IV)}(x)$	⟷ Streckenlast	$\frac{d}{dx}$	← stat. bestimmt oder unbestimmt gelagert



Biegelinie

$$\boxed{w''(x) = \frac{-M_y(x)}{E\, I_{yy}(x)}}$$



$W_{max} = f$ = max. Durchbiegung

EI = Biegesteifigkeit

Biegemoment

$$\boxed{M_y(x) = -E I_{yy}(x)\, w''(x)}$$

Biegelinie $W(x_0) = 0$
Neigung $W'(x_0) = 0$ } Bei Einspannung

$$\boxed{E I w'' = -M_B}$$

$M_B = 0$ wenn Gelenkiges Lager

$$\boxed{E I_y w^{IV}(x) = q(x)}$$

$q(x)$ = Streckenlast

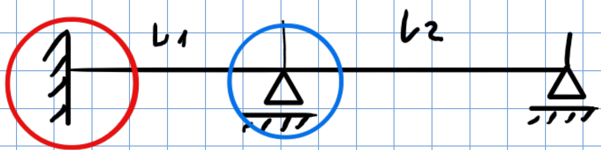
Vorgehen

1. Freischnitt
2. Schnittgrössen bestimmen
3. Biegemomente bestimmen
4. DGL lösen
5. Randbedingungen / Übergangsbedingungen einarbeiten
6. C_i bestimmen und $w(x)$ aufstellen
7. W_{max} bestimmen

Lager	w	w'	M	Q
Gelenkiges Lager	0	$\neq 0$	0	$\neq 0$
Parallelführung	$\neq 0$	0	$\neq 0$	0
Einspannung	0	0	$\neq 0$	$\neq 0$
Freies Ende	$\neq 0$	$\neq 0$	0	0

Randbedingungen

$w(0) = 0,\ w'(0) = 0$



Übergangsbedingung

$w_1(l) = w_2(l)\ ,\ w_1'(l) = w_2'(l)$

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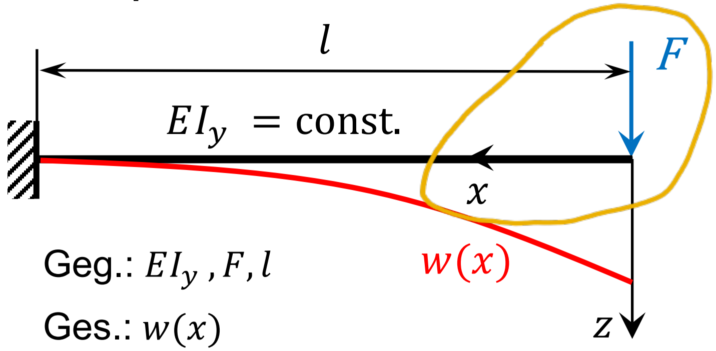
2. Beispiele

Beispiel 1: Kragbalken

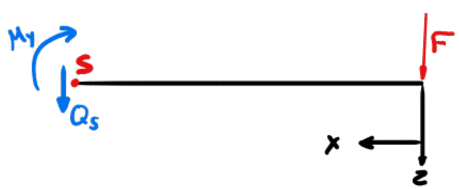


Geg.: EI_y, F, l

Ges.: $w(x)$

1. Biegemoment bestimmen

$$\Sigma M_y = 0: \ M_y^S + Fx = 0 \rightarrow M_y = -Fx$$



2. DGL lösen

M_y einsetzen in DGL
Vorzeichen beachten!

$$w''(x) = -\frac{M_y(x)}{E \cdot I_y} = \frac{Fx}{EI_y}$$

Erste Integration

$$w'(x) = \frac{Fx^2}{2EI_y} + C_1$$

Zweite Integration

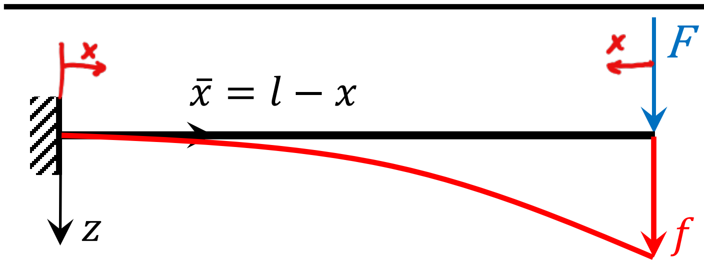
$$w(x) = \frac{Fx^3}{6EI_y} + C_1 x + C_2$$

3. Randbedingungen einarbeiten, C_i bestimmen, dann Biegelinie

$$w'(L) = \frac{FL^2}{2EI_y} + C_1 = 0 \rightarrow C_1 = -\frac{FL^2}{2EI_y}$$

$$w'(L) = \frac{FL^3}{6EI_y} - \frac{Fl^3}{2EI_y} + C_2 = 0 \rightarrow C_2 = \frac{FL^3}{3EI}$$

$$w(x) = \frac{Fx^3}{6EI_y} - \frac{FL^2}{2EI_y} x + \frac{FL^3}{3FI_y} = \frac{FL^3}{6EI_y}\left[\left(\frac{x}{l}\right)^3 - 3\frac{x}{l} + 2\right]$$



Koordinatentransformation

$$w(\bar{x}) = \frac{FL^3}{6EI_y}\left[\left(3\frac{\bar{x}}{l}\right)^2 - \left(\frac{\bar{x}}{l}\right)^3\right]$$

$$\bar{x} = l - x$$

Max. Durchbiegung

$$\boxed{f = w(l) = \frac{FL^3}{3EI_y}}$$

Festigkeitslehre 2
Deformation aufgrund von Biegung

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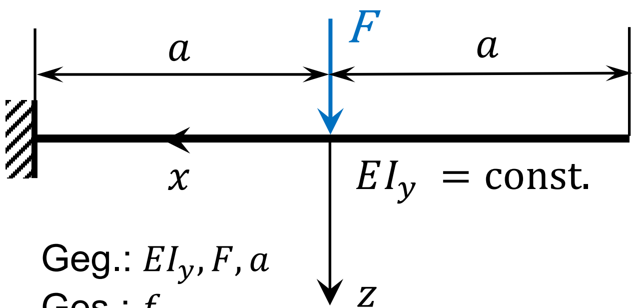
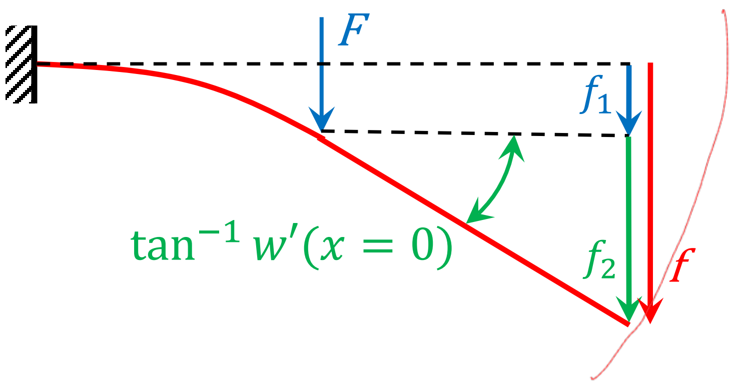
Beispiel 3: Kragbalken mit Starrkörperverschiebung



Geg.: EI_y, F, a
Ges.: f



Aus Beispiel 1:

$$W(x) = \frac{FL^3}{6EI}\left[\left(\frac{x}{l}\right)^3 - 3\frac{x}{l} + 2\right]$$

$$g(x) = \frac{x}{a}$$

$$f(g) = g^3$$

$$\frac{d}{dx}\left(f(g(x))\right) = f' \cdot y' = 3\left(\frac{x}{a}\right)^2 \cdot \frac{1}{a}$$

Man liest ab:

$$f_2 = -a w'(x=0)$$

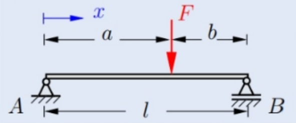
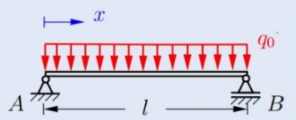
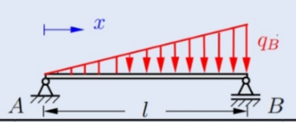
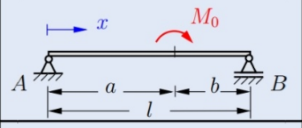
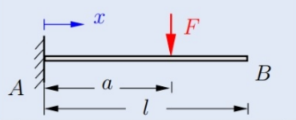
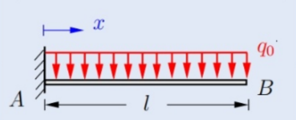
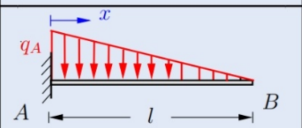
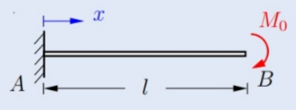
$$f' = 3g^2 \quad y' = \frac{1}{a}$$

Mit $l = a$ aus (1) folgt:

$$w(x) = \frac{Fa^3}{6EI_y}\left[\left(\frac{x}{a}\right)^3 - 3\frac{x}{a} + 2\right], \quad f_1 = w(x=0) = \frac{Fa^3}{3EI_y}$$

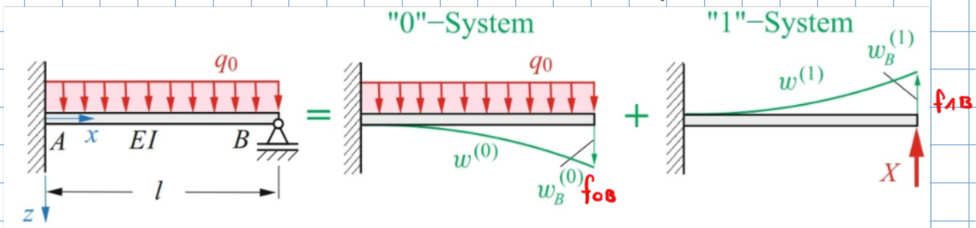
$$w'(x) = \frac{Fa^2}{6EI}\left[3\left(\frac{x}{a}\right)^2 - 3\right]$$

$$f = f_1 + f_2 = \frac{5}{6}\frac{Fa^3}{EI_y}$$

Nr.	Lastfall	EIw'_A	EIw'_B	$EIw(x)$	$EIw_{\max}$
1		$\frac{Fl^2}{6}(\beta-\beta^3)$	$-\frac{Fl^2}{6}(\alpha-\alpha^3)$	$\frac{Fl^3}{6}[\beta\xi(1-\beta^2-\xi^2)+<\xi-\alpha>^3]$	$\frac{Fl^3}{48}$ für $\alpha=\beta=1/2$
2		$\frac{q_0l^3}{24}$	$-\frac{q_0l^3}{24}$	$\frac{q_0l^4}{24}(\xi-2\xi^3+\xi^4)$	$\frac{5}{384}q_0l^4$
3		$\frac{7}{360}q_Bl^3$	$-\frac{1}{45}q_Bl^3$	$\frac{q_Bl^4}{360}(7\xi-10\xi^3+3\xi^5)$	siehe Aufgabe 3.13
4		$\frac{M_0l}{6}(3\beta^2-1)$	$\frac{M_0l}{6}(3\alpha^2-1)$	$\frac{M_0l^2}{6}[\xi(3\beta^2-1)+\xi^3-3<\xi-\alpha>^2]$	$\frac{M_0l^2}{27}\sqrt{3}$ für $a=0$
5		0	$\frac{Fa^2}{2}$	$\frac{Fl^3}{6}[3\xi^2\alpha-\xi^3+<\xi-\alpha>^3]$	$\frac{Fl^3}{3}$ für $a=l$
6		0	$\frac{q_0l^3}{6}$	$\frac{q_0l^4}{24}(6\xi^2-4\xi^3+\xi^4)$	$\frac{q_0l^4}{8}$
7		0	$\frac{q_Al^3}{24}$	$\frac{q_Al^4}{120}(10\xi^2-10\xi^3+5\xi^4-\xi^5)$	$\frac{q_Al^4}{30}$
8		0	M_0l	$M_0\frac{x^2}{2}$	$M_0\frac{l^2}{2}$

Abkürzungen: $\xi=\frac{x}{l}$, $\alpha=\frac{a}{l}$, $\beta=\frac{b}{l}$, $()'\,\widehat{=}\,\frac{\mathrm{d}}{\mathrm{d}x}()=\frac{1}{l}\frac{\mathrm{d}}{\mathrm{d}\xi}()$, $<\xi-\alpha>^n\ \widehat{=}$ Föppl–Klammer.

1. Fach statisch unbestimmtes System



$$f_b = f_{0B} + f_{1B} = 0$$

$$f_B = f_{0B} + f_{1B} = \frac{q_0 l^4}{8 EI_y} + \frac{F_b l^3}{3 EI_y} = 0 \quad \Rightarrow \quad F_B = -\frac{3}{8} q_0 l$$

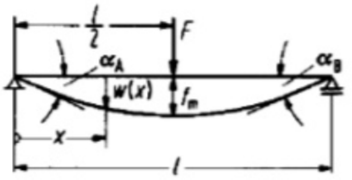
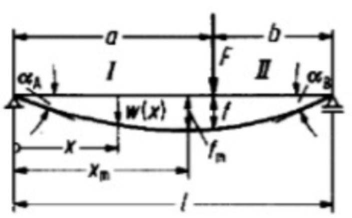
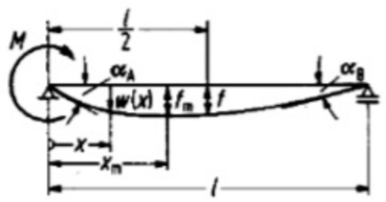
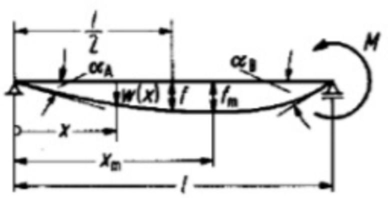
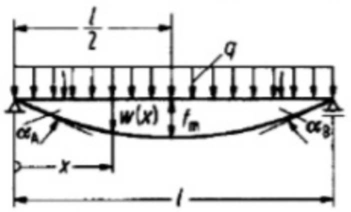
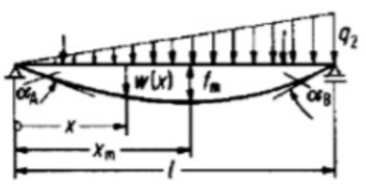
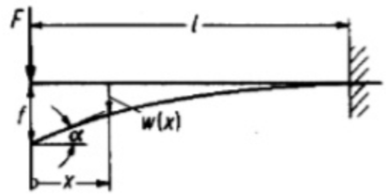
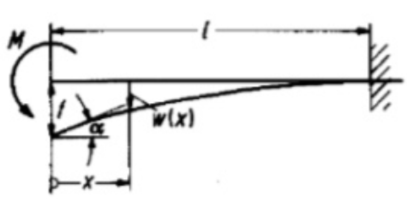
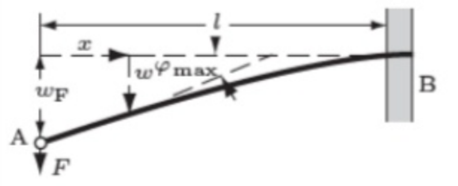
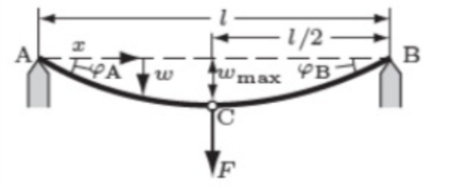
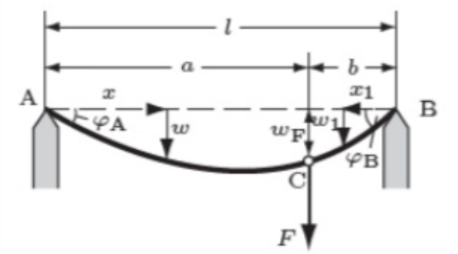
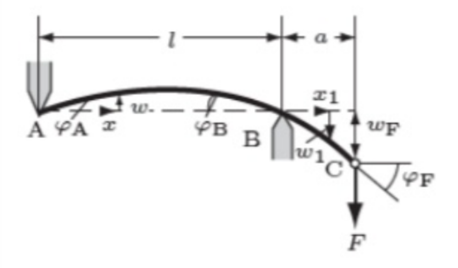
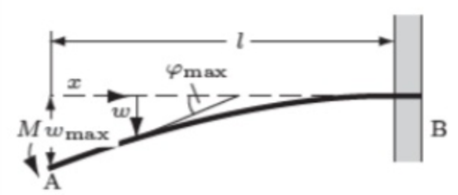
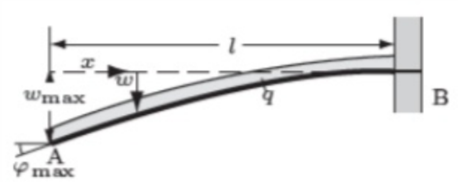
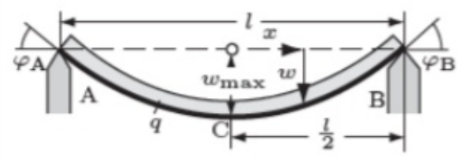
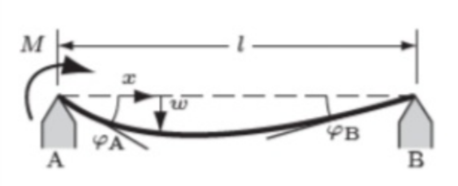
	Belastungsfall	Gleichung der Biegelinie	Durchbiegung	Neigungswinkel
1		$0 \le x \le l/2$: $w(x) = \frac{Fl^3}{48EI_y}\left[3\frac{x}{l} - 4\left(\frac{x}{l}\right)^3\right]$	$f_m = \frac{Fl^3}{48EI_y}$	$\alpha_A = \alpha_B = \frac{Fl^2}{16EI_y}$
2		$0 \le x \le a$: $w_I(x) = \frac{Fab^2}{6EI_y}\left[\left(1+\frac{l}{b}\right)\frac{x}{l} - \frac{x^3}{abl}\right]$ $a \le x \le l$: $w_{II}(x) = \frac{Fa^2b}{6EI_y}\left[\left(1+\frac{l}{a}\right)\frac{l-x}{l} - \frac{(l-x)^3}{abl}\right]$	$f = \frac{Fa^2b^2}{3EI_yl}$ $a>b$: $f_m = \frac{Fb\sqrt{(l^2-b^2)^3}}{9\sqrt{3}\,EI_yl}$ in $x_m = \sqrt{(l^2-b^2)/3}$ $a<b$: $f_m = \frac{Fa\sqrt{(l^2-a^2)^3}}{9\sqrt{3}\,EI_yl}$ in $x_m = l - \sqrt{(l^2-a^2)/3}$	$\alpha_A = \frac{Fab(l+b)}{6EI_yl}$ $\alpha_B = \frac{Fab(l+a)}{6EI_yl}$
3a		$w(x) = \frac{Ml^2}{6EI_y}\left[2\frac{x}{l} - 3\left(\frac{x}{l}\right)^2 + \left(\frac{x}{l}\right)^3\right]$	$f = \frac{Ml^2}{16EI_y}$ in $x = \frac{l}{2}$ $f_m = \frac{Ml^2}{9\sqrt{3}EI_y}$ in $x_m = l - \frac{l}{\sqrt{3}}$	$\alpha_A = \frac{Ml}{3EI_y}$ $\alpha_B = \frac{Ml}{6EI_y}$
3b		$w(x) = \frac{Ml^2}{6EI_y}\left[\frac{x}{l} - \left(\frac{x}{l}\right)^3\right]$	$f = \frac{Ml^2}{16EI_y}$ in $x = \frac{l}{2}$ $f_m = \frac{Ml^2}{9\sqrt{3}EI_y}$ in $x_m = \frac{l}{\sqrt{3}}$	$\alpha_A = \frac{Ml}{6EI_y}$ $\alpha_B = \frac{Ml}{3EI_y}$
4		$w(x) = \frac{ql^4}{24EI_y}\left[\frac{x}{l} - 2\left(\frac{x}{l}\right)^3 + \left(\frac{x}{l}\right)^4\right]$	$f_m = \frac{5}{384}\frac{ql^4}{EI_y}$	$\alpha_A = \alpha_B = \frac{ql^3}{24EI_y}$
5		$w(x) = \frac{q_2l^4}{360EI_y}\left[7\frac{x}{l} - 10\left(\frac{x}{l}\right)^3 + 3\left(\frac{x}{l}\right)^5\right]$	$f_m = \frac{q_2l^4}{153{,}3EI_y}$ in $x_m = 0{,}519\,l$	$\alpha_A = \frac{7}{360}\frac{q_2l^3}{EI_y}$ $\alpha_B = \frac{8}{360}\frac{q_2l^3}{EI_y}$
6		$w(x) = \frac{Fl^3}{6EI_y}\left[2 - 3\frac{x}{l} + \left(\frac{x}{l}\right)^3\right]$	$f = \frac{Fl^3}{3EI_y}$	$\alpha = \frac{Fl^2}{2EI_y}$
7		$w(x) = \frac{Ml^2}{2EI_y}\left[1 - 2\frac{x}{l} + \left(\frac{x}{l}\right)^2\right]$	$f = \frac{Ml^2}{2EI_y}$	$\alpha = \frac{Ml}{EI_y}$

Tabelle 11: Gleichung der Biegelinien für Träger konstanter Biegesteifigkeit

#	Belastungsfall	Gleichung der Biegelinie*)	Durchbiegungen w Winkeländerungen φ*)
1		$w = \frac{Fl^3}{3EI}\left[1 - \frac{3}{2}\cdot\frac{x}{l} + \frac{1}{2}\left(\frac{x}{l}\right)^3\right]$	$w_F = w_{max} = \frac{Fl^3}{3EI}$ $\varphi_{max} = \frac{Fl^2}{2EI}$
2		$w = \frac{Fl^3}{16EI}\cdot\frac{x}{l}\left[1 - \frac{4}{3}\left(\frac{x}{l}\right)^2\right]$ für $x \le \frac{l}{2}$	$w_F = w_{max} = \frac{Fl^3}{48EI}$ $\varphi_A = \varphi_B = \frac{Fl^2}{16EI}$
3		$w = \frac{Fl^3}{6EI}\cdot\frac{a}{l}\cdot\left(\frac{b}{l}\right)^2\cdot\frac{x}{l}\left(1 + \frac{l}{b} - \frac{x^2}{a\cdot b}\right)$ für $x \le a$ $w_1 = \frac{Fl^3}{6EI}\cdot\frac{b}{l}\cdot\left(\frac{a}{l}\right)^2\cdot\frac{x_1}{l}\left(1 + \frac{l}{a} - \frac{x_1^2}{a\cdot b}\right)$ für $x_1 \le b$	$w_F = \frac{Fl^3}{3EI}\cdot\left(\frac{a}{l}\right)^2\cdot\left(\frac{b}{l}\right)^2$ $\varphi_A = w_F\cdot\frac{1}{2a}\left(1 + \frac{l}{b}\right)$ $\varphi_B = w_F\cdot\frac{1}{2b}\left(1 + \frac{l}{a}\right)$
4		$w = \frac{Fl^3}{6EI}\cdot\frac{a}{l}\cdot\frac{x}{l}\left[1 - \left(\frac{x}{l}\right)^2\right]$ für $x \le l$ $w_1 = \frac{Fl^3}{6EI}\cdot\frac{x_1}{l}\left[\frac{2a}{l} + 3\frac{a}{l}\cdot\frac{x_1}{l} - \left(\frac{x_1}{l}\right)^2\right]$ für $x_1 \le a$	$w_F = \frac{Fl^3}{3EI}\cdot\left(\frac{a}{l}\right)^2\left(1 + \frac{a}{l}\right)$ $\varphi_A = \frac{Fl^2}{6EI}\cdot\frac{a}{l} = \frac{1}{2}\varphi_B$ $\varphi_F = \frac{Fl^2}{6EI}\cdot\frac{a}{l}\cdot\left(2 + 3\frac{a}{l}\right)$
5		Kreisbogen mit dem Radius $\varrho = \frac{EI}{M}$ Näherungsweise $w = \frac{Ml^2}{2EI}\left(1 - \frac{x}{l}\right)^2$	$w_F = \frac{Ml^2}{2EI}$ $\varphi_{max} = \frac{Ml}{EI}$
6		$w = \frac{ql^4}{8EI}\left[1 - \frac{4}{3}\cdot\frac{x}{l} + \frac{1}{3}\left(\frac{x}{l}\right)^4\right]$	$w_{max} = \frac{ql^4}{8EI}$ $\varphi_{max} = \frac{ql^3}{6EI}$
7		$w = \frac{5ql^4}{384EI}\left[1 - 4\left(\frac{x}{l}\right)^2\right]\left[1 - \frac{4}{5}\left(\frac{x}{l}\right)^2\right]$	$w_{max} = \frac{5ql^4}{384EI}$ $\varphi_A = \varphi_B = \frac{ql^3}{24EI}$
8		$w = \frac{M_A l^2}{3EI}\left[\frac{x}{l} - \frac{3}{2}\left(\frac{x}{l}\right)^2 + \frac{1}{2}\left(\frac{x}{l}\right)^3\right]$	$w_{max} = \frac{M_A l^2}{15{,}59EI}$ bei $x = 0{,}423l$ $\varphi_A = \frac{M_A l}{3EI} = 2\varphi_B$

*) Die Gleichungen gelten für $(w')^2 << 1$

Torsion

Schubspannung

$$\boxed{\tau = \frac{M_T}{I_T} \cdot r}$$

$$\boxed{M_T = \frac{P \cdot 9550}{n}} = Nm$$

kW

1/min

Torsionswiderstandsmoment

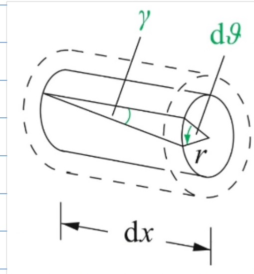
$$\boxed{W_T = \frac{I_T}{R}}$$

$$\boxed{W_T = \frac{\pi D^3}{16}}$$ voller Rundquerschnitt

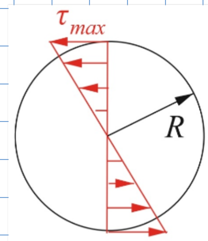
G = Schubmodul

$$\boxed{\tau = G \cdot r \cdot \frac{d\vartheta}{dx}}$$

Verwindung



τ wächst linear nach aussen, weil Verwindung im QS konstant



Torsionsträgheitsmoment eines Kreises

$$\boxed{I_{T\,Kreis} = \pi \frac{R^4}{2} = \frac{\pi D^4}{32}}$$

Torsionsträgheitsmoment eines Kreisringes

$$\boxed{I_{T\,Kreisring} = \frac{\pi}{2}\left(R_a^4 - R_i^4\right) = \frac{\pi}{32}\left(d_a^4 - d_i^4\right)}$$

Endverdrehung (Torsionswinkel)

$$\boxed{\vartheta_i = \frac{M_T \cdot l}{G \cdot I_T}} \text{ [rad]}$$

Nmm, mm, MPa, mm⁴

Nur wenn I_T konstant, wenn kegel → integral $\vartheta(x) = \int_0^x \frac{M_T}{G \cdot I_T(x)}\,dx$

$$grad = rad \cdot \frac{180°}{\pi}$$

G in Pa MPa $\xrightarrow{\cdot 10^6}$ Pa

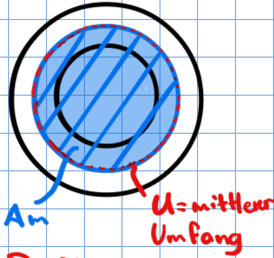
Torsion dünnwandiger geschlossener Profile

Erste Formel von Bredt

$$\boxed{\tau = \frac{M_T}{2A_m t} = \frac{M_T}{W_T}}$$

Torsionswiderstandsmoment

$$\boxed{W_T = 2A_m t_{min}}$$

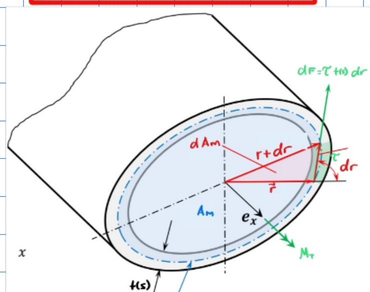


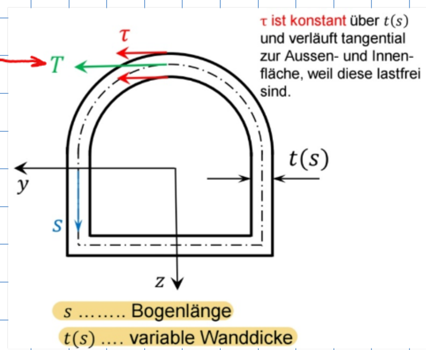
Nur bei dünnwandigen Profilen

Dünnwandig wenn: $\frac{t}{r_m} < 0.1$

Schubfluss

$$\boxed{T = \tau(s) \cdot t(s)}$$





Formänderung

Zweite Formel von Bredt

$$\boxed{I_T = \frac{4A_m^2}{\oint \frac{1}{t}\,ds}}$$

τ_{max} bei geschlossenen Profil an min. Wanddicke; bei offenen Profil an max. Wanddicke

Torsionsteif. konst. Dicke

$$\boxed{I_T = \frac{4A_m^2 t}{u}}$$

Wanddicke t = konst.

U = Umfang der QS Dicke

Torsion dünnwandiger offener Profile

t ist immer das dünnere Stück!

Torsionswiderstandsmoment eines schmalen Rechteckquerschnittes

$$\boxed{W_T = \frac{1}{3} h t^2}$$

Torsionsträgheitsmoment eines schmalen Rechteckquerschnittes

$$\boxed{I_T = \frac{1}{3} h t^3}$$

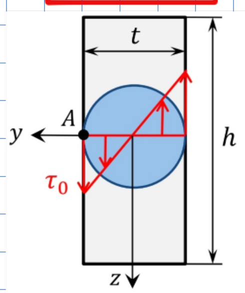
Torsionsträgheitsmoment eines aus schmalen Rechtecken zusammengesetzten Querschnittes:

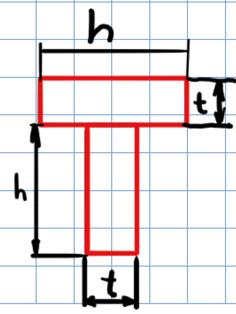
$$\boxed{I_T = \frac{1}{3} \sum h_i \cdot t_i^3}$$

Torsionswiderstandsmoment eines aus schmalen Rechtecken zusammengesetzten Querschnittes

$$\boxed{W_T = \frac{1}{3 t_{max}} \sum h_i \cdot t_i^3}$$

$$\boxed{\tau(y) = \frac{2y}{t} \tau_0}$$





Querkraftschub $Q(x) = F$

Genietet, geschraubt, geklebte Träger

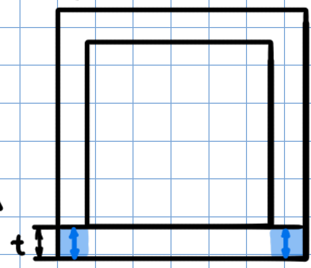
$$\boxed{\tau_{xz}(x,z) = \frac{Q(x)\,S(z)}{I_y\, b(z)}}$$

Statisches Moment der Restfläche

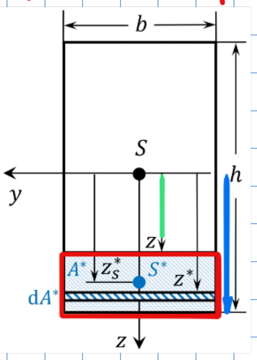
$$\boxed{S(z) = \int_A z\, dA^*}$$ A^* bzg. Svp. $z=0$

mm^3

$b(z) = 2t$



Bsp. Rechteckquerschnitt



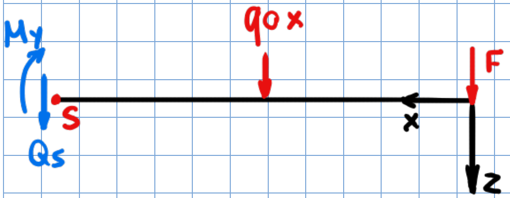
Statisches Moment

$$S(z) = \int_A z^* dA^* = \int_z^{\frac{h}{2}} z^* b\, dz = b\left[\frac{z^{*2}}{2}\right]_z^{\frac{h}{2}} = \underline{\underline{b\left(\frac{h^2}{8} - \frac{z^2}{2}\right)}}$$

Schubspannung

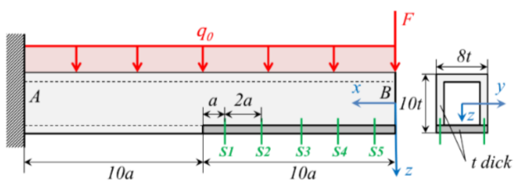
$$\tau_{xy}(z) = \frac{Q(x)\,S(z)}{I_y\, b(z)} = \frac{Qb\left(\frac{h^2}{8} - \frac{z^2}{2}\right)}{\frac{bh^3}{12}\cdot b} = \underline{\underline{\frac{Q}{bh^3}\left(\frac{3}{2}h^2 - 6z^2\right)}}$$

Biegemoment



Aufgabe 23

Bei einem in *A* fest eingespannten Träger mit durchgehendem Rechteckhohlprofil (Wanddicke *t*) untersuche man, ob aus Wartungsgründen die vordere Hälfte der unteren Profilwand mit nur 5 Schraubenpaaren *S1* bis *S5* (*x*-Abstand jeweils *2a*) befestigt werden kann. Als Belastung wirkt die konstante Streckenlast q_0 sowie in *B* die Kraft *F*. Für *F* = $100\, q_0 t$ und $a = 5t$ ermittle man:



1. Den Verlauf des Biegemomentes $M_b = M_b(q_0, x, t)$ und der Querkraft $Q = Q(q_0, x, t)$ für das eingezeichnete Koordinatensystem.
2. Die in der Fuge ($0 < x < 10a$, $z = 4t$) von den Schraubenverbindungen zu übertragende Schubspannung $\tau = \tau(q_0, x, t)$.
3. Das Schraubenpaar, das die kleinste Schubspannung übertragen muss.
4. Für das Paar *S1* die notwendige Vorspannkraft $F_V = F_V(q_0, t)$ pro Schraube, damit diese <u>nicht</u> auf Schub beansprucht wird. Der Haftungskoeffizient in der Fuge beträgt $\mu = \frac{1}{10}$.

Querkraft Q

$$\Sigma F\uparrow: -Q_s - q_0 x - F = 0 \rightarrow Q_s(x) = -q_0 x - F = \underline{\underline{-100 q_0 t - q_0 x}}$$

Biegemoment M

$$\Sigma M_s: -M_y - q_0 x \cdot \frac{1}{2}x - Fx = 0 \rightarrow M_y = -\frac{1}{2}q_0 x^2 - Fx = \underline{\underline{-\frac{1}{2}q_0 x^2 - 100 q_0 t x}}$$

b) $$\tau(q_0, x, t) = \frac{Q(x)\,S(z)}{I_y\, b(z)} \qquad S(z) = \int_{4t}^{5t} 4.5t \cdot 8t\, dz = \left[4.5t \cdot z \cdot 8\right]_{4t}^{5t} = 36t^3$$

$$I_y = \frac{8t(10t)^3}{12} - \frac{6t(8t)^3}{12} = \frac{1232}{3}t^4 \qquad B(z=4) = 2t$$

$$\tau(q_0, x, t) = -\frac{(-q_0 x - 100 q_0 t)\cdot 36t^3}{\frac{1232}{3}t^4 \cdot 2t} = \underline{\underline{-\frac{27(-q_0 x - 100 q_0 t)}{4000 t^4}}}$$

4. Vorspannkraft

$$Q = \int_{8a}^{10a} \tau \cdot b(x)\, dx = \frac{27t}{616t^2}\int_{8a}^{10a} -q_0(100t + x)\, dx = -64 q_0 t \qquad F_v > \frac{|Q_{Fuge}|}{\mu} = \frac{64 q_0 t}{0.1} = \underline{\underline{640 q_0 t}}$$