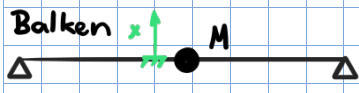
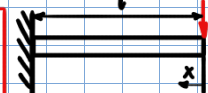


Freie gedämpfte Schwingung f=1 + Spannungsanalyse

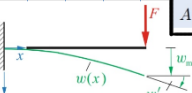


$$I = \frac{bh^3}{12}$$



$$M_{max} = F \cdot l$$

$$w = \frac{Fl^3}{3EI}$$



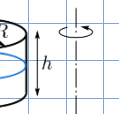
$$f = \frac{FL^3}{48EI}$$

$$C_{Balken} = \frac{48EI}{L^3}$$

Scheibe ($h/R \ll 1$), Zylinder

$$\Theta_S = \frac{1}{2}mR^2$$

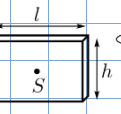
$$\Theta_P = \frac{3}{2}mR^2$$



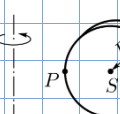
Stab ($h/l \ll 1$), Platte

$$\Theta_S = \frac{1}{12}ml^2$$

$$\Theta_P = \frac{1}{3}ml^2$$



Scheibe



$$\Theta_S = \frac{1}{4}mR^2$$

$$\Theta_P = \frac{5}{4}mR^2$$

Doppel:

$$M_{max} = \frac{F \cdot l}{4}$$

$$\sigma_{max} = \frac{M \cdot h}{I \cdot 2} = \frac{F \cdot l \cdot h}{\left(\frac{bh^3}{12}\right) \cdot 2} = \frac{6Fl}{bh^2}$$

1. Bewegungsgleichung bestimmen (Impuls, Spinsätze, Kraftgesetz, Kinematikskizze) $\rightarrow 7m\ddot{x} + 9dx + 9cx = 0$

2. Eigenkreisfrequenz $\omega_0 = \sqrt{\frac{c}{m}} = \sqrt{\frac{9c}{7m}}$ Dämpfungswert $\delta = \frac{d}{2m}$ 3. Lehrsche Dämpfung $D = \frac{\delta}{\omega_0} = \frac{d}{2\sqrt{c \cdot m}}$

$D = 0$: Ungedämpfte Schwingung: $y(t) = A \cdot \cos(\omega_0 t) + B \cdot \sin(\omega_0 t)$ $\dot{y}(t) = -A \cdot \sin(\omega_0 t) \cdot \omega_0 + B \cdot \cos(\omega_0 t) \cdot \omega_0$ $T_0 = \frac{1}{f_0} = \frac{2\pi}{\omega_0}$

Bsp. $z(0) = A = -y_0$, $\dot{z}(0) = B \cdot \omega_0 = 0 \rightarrow z(t) = -y_0 \cdot \cos(\omega_0 t)$ $m\ddot{x} + d\dot{x} + cx = 0 \rightarrow \ddot{x} + 2D\omega_0\dot{x} + \omega_0^2 x = 0$ $\omega_n = \sqrt{\frac{c}{m}}$
Annahme bei $t=0$ aus entspannter Lage stoßfrei losgelassen

$0 < D < 1$: Unterkritisch gedämpfte Schwingung: $\omega = \omega_0^2 - \delta^2 = \omega_0^2(1 - D^2)$, $\omega = \sqrt{\omega_0^2 - \left(\frac{d}{2m}\right)^2}$ $D_{neu} = \frac{d_{alt}}{d_{neu}}$

$$y(t) = e^{-\delta t} (A_1 \cdot \cos(\omega t) + A_2 \cdot \sin(\omega t))$$

$$\dot{y}(t) = e^{-\delta t} [(\omega A_2 - \delta A_1) \cos(\omega t) - (\omega A_1 + \delta A_2) \sin(\omega t)]$$

$D = 1$: Kritisch gedämpft $y(t) = A_1 \cdot e^{-\delta t} + A_2 \cdot t \cdot e^{-\delta t}$ $\dot{y}(t) = -A_1 \cdot e^{-\delta t} \cdot \delta + A_2 \cdot e^{-\delta t} - \delta A_2 \cdot t \cdot e^{-\delta t}$

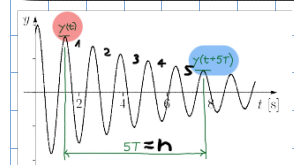
Bsp. $y(0) = y_0 \rightarrow y(0) = y_0 = A_1$, $\dot{y}(0) = 0 = -\delta A_1 + A_2 \rightarrow A_2 = \delta A_1$, $y(t) = y_0(1 + \delta t)e^{-\delta t}$

$D > 1$: Überkritisch gedämpft $\Psi^2 = \delta^2 - \omega_0^2 = \omega_0^2(D^2 - 1)$ $D \rightarrow \infty$: Grenzfall $\frac{\Psi}{\omega_0} = \sqrt{D^2 - 1} - \sqrt{D^2}$

$$y(t) = e^{-\delta t} (A \cdot \cosh(\Psi t) + B \cdot \sinh(\Psi t))$$

$$\dot{y}(t) = e^{-\delta t} [(B\Psi - \delta A) \cosh(\Psi t) + (A\Psi - \delta B) \sinh(\Psi t)]$$

Logarithmisches Dekrement



$$\Delta = 2\pi \sqrt{\frac{D^2}{1 - D^2}}$$

$$\Delta = \frac{1}{n} \ln \left(\frac{y_k}{y_{k+n}} \right)$$

$$D = \sqrt{\frac{\Delta^2}{\Delta^2 + 4\pi^2}}$$

$$\Delta = \delta \cdot T = \delta \cdot \frac{2\pi}{\omega}$$

$$y(0) = -y_0 \rightarrow A = -y_0$$
, $\dot{y}(0) = v_0 = B\Psi - \delta A \rightarrow B = \frac{v_0 + \delta A}{\Psi} = \frac{v_0 - y_0 \delta}{\Psi}$

$$\rightarrow y(t) = e^{-\delta t} \left(\frac{v_0 - y_0 \delta}{\Psi} \cdot \sinh(\Psi t) - y_0 \cdot \cosh(\Psi t) \right)$$

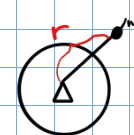
$$\text{Resonanzdrehzahl } n = \frac{\omega_0 \cdot 60}{2\pi}$$

$$\text{Periodendauer } T = \frac{2\pi}{\Omega}$$

$$n = \frac{\Omega \cdot 60}{2\pi} \rightarrow \Omega = \frac{2\pi n}{60}$$

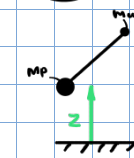
Frequenzverhältnis $\eta = \frac{\Omega}{\omega_0}$ $\Omega = \text{Anregung (Motordrehzahl)}$ $\omega_0 = \text{Eigenkreisfrequenz Ersatz}$

Erregte ungedämpfte Schwingung f=1 rotierende Ummwucht



$$F_u \text{ gegeben} = m \cdot r \cdot \Omega^2 \cdot \sin(\Omega t)$$

$$(m_p + m_u) \ddot{x} + c x = F_u$$



$$z_u = z + r \cdot \sin(\Omega t)$$

$$\ddot{z}_u = \ddot{z} - r \cdot \Omega^2 \cdot \sin(\Omega t)$$

$$F_u = m_u \ddot{z}_u = m_u \ddot{z} - m_u \Omega^2 r \cdot \sin(\Omega t)$$

$$m_p \ddot{z} = -F_u - F_c = -m_u \ddot{z} + m_u \Omega^2 r \cdot \sin(\Omega t) - c \cdot z$$

$$(m_p + m_u) \ddot{z} + c \cdot z = m_u \Omega^2 r \cdot \cos(\Omega t)$$

Partikuläre Lösung für kleine Auslenkungen

$$\omega_0 = \frac{\Omega}{\eta}$$

$$\omega_0 = \sqrt{\frac{c}{m}} \rightarrow c^* = m \cdot \omega_0^2$$

$$M_{max} = F_{max} \cdot l = c \cdot x_{max} \cdot l$$

$$x_{max} = \frac{m_u r \Omega^2}{|c^* - m \Omega^2|} = \frac{m_u r \eta^2}{m^* |1 - \eta^2|}$$

$$F_{min} = m g - F_{Feder}$$
 Keine Zugkräfte $\rightarrow x_{max} \leq \frac{m g}{c^*}$

$$m c \leq \frac{15}{16} \frac{m g}{\left(\frac{\pi n}{120}\right)^2 \cdot e}$$

$$m \cdot \ddot{z} = -F_c - F_d$$

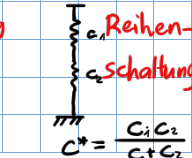
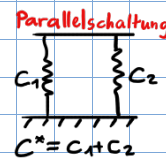
$$m \cdot (\ddot{x} + \ddot{x}_G) = -F_c - F_d$$

$$m \ddot{x} + d \dot{x} + c x = -\ddot{x}_G \cdot m$$

$$m \ddot{x} + d \dot{x} + c x = m x_0 \Omega^2 \cos(\Omega t)$$

$$x_G = x_0 \cdot \cos(\Omega t)$$

$$\ddot{x}_G = -x_0 \Omega^2 \cos(\Omega t)$$



Keine Zugkräfte $\rightarrow x_{max} = \frac{m \cdot g}{c^*}$
 $F_{min} = m g - x_{max} \cdot c^*$

Vergrößerungsfunktion:

$$V_F = \frac{1}{\sqrt{(1 - \eta^2)^2 + 4D^2 \eta^2}}$$

$$V_W = \frac{\eta^2}{\sqrt{(1 - \eta^2)^2 + 4D^2 \eta^2}}$$

$$V_{max} = \frac{1}{2D \sqrt{1 - D^2}}$$

Freie ungedämpfte Schwingungen $f=2$

1. Freischnitt
2. Kinematische Relation
3. Kraftgesetz Federn
4. Impuls+Spinsätze

↑ Rolle: $M\ddot{x} = F_k + Mg - F_c - F_1$

Rolle ↓: $\ddot{\varphi} \theta_s = R F_1 - R F_c$

$x_h = x - h\varphi$
 $x_v = x + v\varphi$
 $\sin(\varphi) = \varphi$ Kleine Winkel

↑ Masse: $m\ddot{y} = mg - F_k$

Gl.1 $(M + \frac{\theta_s}{R^2})\ddot{x} + 4cx + kx - ky = Mg$

Gl.2 $my + ky - kx = mg$

$$\begin{pmatrix} M + \frac{\theta_s}{R^2} & 0 \\ 0 & m \end{pmatrix} \begin{pmatrix} \ddot{x} \\ \ddot{y} \end{pmatrix} + \begin{pmatrix} 4c+k & -k \\ -k & +k \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} Mg \\ mg \end{pmatrix}$$
 Entkoppelt

Gekoppelt, wenn Beide Koordinaten in einer Zeile

Wenn $\begin{pmatrix} m\lambda^2 + 2c & 0 \\ 0 & \theta\lambda^2 + \frac{1}{2}lc \end{pmatrix} \rightarrow (m\lambda^2 + 2c)(\theta\lambda^2 + \frac{1}{2}lc) = 0$
 $m\lambda^2 + 2c = 0 \rightarrow \lambda_1^2 = -\frac{2c}{m} \rightarrow \omega_1 = \sqrt{\frac{2c}{m}}$

MK-System

$$\begin{pmatrix} 3m & 0 \\ 0 & 2m \end{pmatrix} \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{pmatrix} 2c & -c \\ -c & c \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ cl \end{pmatrix}$$

$M \cdot \ddot{x} + K \cdot x = F$

Ansatz Homogene $(M\lambda^2 + K)u = 0$

Eigenwerte $0 = \det(M\lambda^2 + K) = \det \begin{pmatrix} 3m\lambda^2 + 2c & -c \\ -c & 2m\lambda^2 + c \end{pmatrix} = 0$

$\rightarrow 6m^2\lambda^4 + 7mc\lambda^2 + c^2 = 0$

$\omega = \sqrt{-\lambda^2}$ $\lambda^2 = -\omega^2$

$\lambda_{1,2,3,4} = \frac{-7mc \pm \sqrt{49m^2c^2 - 4 \cdot 6m^2 \cdot c^2}}{2 \cdot 6m^2}$

Matrix immer Maximal vereinfachen!

$$\begin{pmatrix} 2m l^2 & m l^2 \\ m l^2 & m l^2 \end{pmatrix} \begin{pmatrix} \ddot{\alpha} \\ \ddot{\beta} \end{pmatrix} + \begin{pmatrix} 2mg l & 0 \\ 0 & mg l \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$m l^2 \left(\begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \ddot{\alpha} \\ \ddot{\beta} \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} \right) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $q = \frac{mg l}{m l^2} = \frac{g}{l}$

Eigenvektoren: $(M\lambda^2 + K)u = 0$

$\lambda_{1,3}^2: \begin{pmatrix} 3m(-\frac{c}{6m}) + 2c & -c \\ -c & 2m(-\frac{c}{6m}) + c \end{pmatrix} = \begin{pmatrix} -\frac{c}{2} + 2c & -c \\ -c & -\frac{c}{3} + c \end{pmatrix} = \begin{pmatrix} \frac{3c}{2} & -c \\ -c & \frac{2c}{3} \end{pmatrix}$

Nur 1 Zeile benötigt!

$I = \frac{bh^3}{12} = \frac{0,05m \cdot (0,1m)^3}{12}$

$E = 210000 MPa = 210 \cdot 10^9 Pa$

$C^* = \frac{6EI}{L^3} + 2c$

$C = 10 N/mm = 10000 N/m$

$m^* = m_p + m_u$

$\omega_0 = \sqrt{\frac{C^*}{m^*}} = \sqrt{\frac{\frac{6E \cdot \frac{bh^3}{12}}{L^3} + 2c}{m_p + m_u}} = \left[\frac{rad}{s} \right]$

$\begin{pmatrix} \frac{3c}{2} & -c \\ -c & \frac{2c}{3} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \rightarrow \frac{3c}{2} \cdot u_1 - c \cdot u_2 = 0 \quad | :c$
 $\frac{3}{2} u_1 = u_2 \rightarrow u_1 = \begin{pmatrix} 1 \\ \frac{3}{2} \end{pmatrix}$

$\lambda_{2,4}^2: \begin{pmatrix} 3m(-\frac{c}{m}) + 2c & -c \\ -c & 2m(-\frac{c}{m}) + c \end{pmatrix} = \begin{pmatrix} -3c + 2c & -c \\ -c & -2c + c \end{pmatrix} = \begin{pmatrix} -c & -c \\ -c & -c \end{pmatrix}$

$\begin{pmatrix} -c & -c \\ -c & -c \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} \rightarrow -cu_1 - cu_2 = 0 \quad | :c$
 $-u_1 = u_2 \rightarrow u_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

Auch nach u_1 Normieren aber $\ddot{u}_2$ schreiben, weil anderes λ^2 verwendet!

$$\left[\frac{rad}{s} \right] = \sqrt{\frac{\frac{6 \cdot 210 \cdot 10^9 Pa \cdot \frac{0,05m \cdot (0,1m)^3}{12}}{(5m)^3} + 2 \cdot 10000 N/m}{50kg + 0,6kg}}$$

Wenn $\begin{pmatrix} 0 & 1 \\ 0 & -\frac{1}{2} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = 0 \rightarrow u_1 = \begin{pmatrix} \lambda \\ 0 \end{pmatrix} \lambda \in \mathbb{R}$